Quiz 3 Solution

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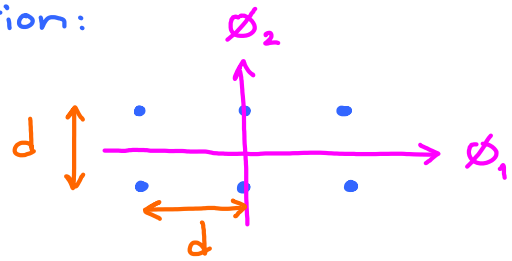
Express $\frac{d}{2\sigma}$ in the form of $\frac{E_b}{N_0}$ for the following constellation:



Assume equiprobable message.

$$E_s = \frac{1}{6}\left(2\times\left(\frac{d}{2}\right)^2 + 4\times\left(\left(\frac{d}{2}\right)^2 + d^2\right)\right) = \frac{1}{6}\left(\frac{d^2}{2} + 2\times\frac{5d^2}{4}\right) = \frac{11}{12}d^2$$

$$E_b = \frac{1}{\log_2 6}\frac{11}{12}d^2 \quad \Rightarrow d^2 = \frac{12}{11}(\log_2 6)E_b$$

$$\frac{d}{2\sigma} = \sqrt{\frac{d^2}{4\sigma^2}} = \sqrt{\frac{d^2}{2N_0}} = \sqrt{\frac{6}{11}(\log_2 6)\frac{E_b}{N_0}}$$

Therefore, for the constellation above,

$$P(\varepsilon) = \frac{7}{3}q_f - \frac{4}{3}q_f^2 \quad \text{where} \quad q_f = Q\left(\sqrt{\frac{6}{11}(\log_2 6)\frac{E_b}{N_0}}\right)$$